

L-Moment based Flood Frequency Modelling

R K Jaiswal, *Associate Member*

N K Goel, *Non-member*

P Singh, *Non-member*

T Thomas, *Non-member*

Extreme environmental events such as floods cause a lot of damage to life and property of human society. A reliable estimation of magnitude and frequency of occurrence of such extreme events is of great significance in minimizing damages by facilitating proper planning and design of civil engineering structures such as bridges, barrages and dams. In the present study, the L-moments have been used for flood frequency modelling of Beas basin. River Beas, which is an important river of Indus river system, has been selected for the study. The Beas basin is frequently affected by flash floods in the main river and its tributaries. Annual flood series of 20 years or so of eight sites in the basin have been used for the study. The homogeneity has been verified using tests based on L-moments. The EV-I, generalised extreme value (GEV), logistic, generalised logistic, generalised pareto, normal and lognormal distributions have been applied for flood frequency modelling. From the goodness of fit tests based on L-moments coupled with simulations, it has been found that the GEV distribution can be used for estimation of probable floods for Beas basin.

Keywords: L-moments; Homogeneity; Return period; Quantile distributions

INTRODUCTION

Estimation of design flood is of great importance for variety of hydraulic structures such as weirs, barrages, dams, spillways and bridges. There are two basic approaches for determination of designed peak flood, namely, deterministic and probabilistic approach. The probabilistic approach is based on the frequency analysis of past flood records and requires lesser data as compared to the deterministic approach. In flood frequency modelling the sample data (usually annual maximum series) is used to fit frequency distributions, which in turn are used to extrapolate from record events to design events. The present study has been taken up to develop flood frequency formula for the Beas basin. In the analysis, *L*-moments have been used for establishment of homogeneity, parameter estimation and determination of best-fit distribution.

REVIEW OF LITERATURE

Statistical flood frequency analysis is one of the most active areas of research. The abstracts of different researches in the field of homogeneity tests and regional flood frequency analysis have been described here.

Homogeneity Tests

In the regional flood frequency modelling various sites of a region are grouped together for estimation of regional

R K Jaiswal and T Thomas are with the Ganga Plains South Regional Centre, National Institute of Hydrology, Sagar 470 001, Dr N K Goel is with the Department of Hydrology, IIT Roorkee, Roorkee 247 667, while Dr P Singh is with National Institute of Hydrology, Roorkee, Roorkee 247 667.

This paper was received on July 17, 2002. Written discussion on the paper will be received until July 31, 2003.

parameters. Initially, Dalrymple provided homogeneity test to check homogeneity of a region. Wiltshier¹ suggested some drawbacks of Dalrymple tests. Hosking and Wallis² recommended a homogeneity test based on *L*-moment ratio. Fill and Stedinger³ analyzed the Dalrymple's homogeneity test and proposed a revised version of Dalrymple test.

Regional Flood Frequency Analysis

Dalrymple⁴ described an index flood technique (USGS method) to carry out regional flood frequency analysis. Benson⁵ pointed out deficiencies in the index flood method proposed by Dalrymple and suggested many modifications in the method. The National Environment Research Council of UK gave a method for regional flood frequency analysis based on order statistics in 1975⁶.

Wallis⁷ recommended a method based on standardized probability weighted moments for regional flood frequency analysis. Landwehr, *et al*^{8,9}, Kuczera¹⁰, Gries and Wood¹¹, Hosking¹², Hosking and Wallis^{13,14}, Singh¹⁵, Rakesh Kumar, *et al*¹⁶ and several others have investigated various issues involved in the regional flood frequency modelling.

STUDY AREA

The river Beas, one of the important rivers of the Indus river system, has been selected for the study. It originates from the eastern slopes of Rohtang pass of Himalayas at an elevation of 4000 m and flows in nearly north-south direction up to Larji, where it nearly takes a right angle turn towards west and flows in the same direction up to Pandoh. The catchment area of the Beas basin up to Pandoh dam is 5278 km². Some of the major tributaries which join upstream of Pandoh dam are: Parvati river near Bhuntar, Tirthan and Sainj rivers near Larji,



Figure 1 Map showing the Beas basin up to Pandoh dam

Sabari *nala* near Kulu and Bakhli *khad* near Pandoh dam. The drainage map of the river Beas up to Pandoh has been presented in Figure 1.

DATA USED

The 20-years annual peak flood series from 1979 to 1998 of river Larji at Sainj, river Bakhli at Pandoh, river Beas at Manali, river Parvati at Bhunjar, river Beas at Thalout, river Tirthan at Larji and river Beas at Bhunjar have been used for flood frequency modelling. The common statistics of the annual flood series of the sites in the basin have been presented in Table 1.

METHODOLOGY

Methodology for the flood frequency modelling may include testing of homogeneity, development of relationship between mean annual peak flood and catchment characteristics, determination of regional parameters for various distributions, selection of the best-fit distribution and lastly the development of flood frequency formula/curve for the region. In the present study, standardized *L*-moments have been used for developing the flood frequency formula for the Beas basin.

Table 1 Statistics of annual flood series of various sites in the Beas basin upto Pandoh

Statistical Parameters	Pandoh	Thalout	Parvati	Sainj	Bakhli	Tirthan	Bhunjar	Manali
Catchment Area, km ²	5278.000	3444.140	1707.540	723.740	268.770	691.810	2858.360	233.740
Original Series								
Mean	1514.430	1145.499	372.171	206.358	70.243	247.903	839.864	137.787
Standard Deviation	775.608	504.345	132.059	102.479	34.880	151.802	366.942	53.290
Variance	6.01E+5	2.54E+5	1.74E+4	1.05E+4	1.22E+3	2.30E+4	1.35E+5	2.84E+3
C_v	0.503	0.440	0.355	0.497	0.497	0.612	0.437	0.387
C_s	1.267	1.408	0.564	1.617	0.531	1.007	1.328	0.999
C_k	0.750	1.578	-0.119	2.580	-0.540	-0.496	0.563	1.355
r_1	-0.019	-0.035	-0.051	-0.057	-0.444	-0.217	-0.313	0.443
Log Transformed Series								
Mean	7.236	6.695	5.857	5.231	4.122	0.559	6.658	4.858
Standard Deviation	0.457	0.394	0.374	0.445	0.545	0.559	0.380	0.376
Variance	0.209	0.155	0.140	0.198	0.297	0.313	0.145	0.141
C_v	0.063	0.057	0.064	0.085	0.132	0.104	0.057	0.077
C_s	0.488	0.607	-0.554	0.360	-0.402	0.384	0.943	0.126
C_k	-0.315	-0.037	1.276	0.595	-0.632	-0.949	-0.357	-0.539
r_1	0.060	0.016	-0.119	-0.048	-0.401	-0.133	-0.393	0.479

L-moments

Hosking first introduced the L -moments are linear combinations of probability weighted moments (PWMs) and can be obtained as:

$$\lambda_1 = M_{100} \quad (1)$$

$$\lambda_2 = 2M_{110} - M_{100} \quad (2)$$

$$\lambda_3 = 6M_{120} - 6M_{110} + M_{100} \quad (3)$$

$$\lambda_4 = 20M_{130} - 30M_{120} + 12M_{110} - M_{100} \quad (4)$$

It can be seen that the L -moments can be estimated directly from the PWMs. However, the L -moments are more convenient, as they are directly interpretable as measures of the scale and shape of probability distributions. L -moments can be used to estimate parameters when fitting a distribution to a sample, by equating the first p -sample L -moments to corresponding population L -moments. Parameter estimation with L -moments has been found more accurate than even the maximum likelihood estimate, in case of small sample¹². In the present study EV-I, GEV, logistic, generalised logistic, generalised pareto distribution, normal and lognormal distributions using L -moments have been used.

Homogeneity Tests

For testing homogeneity, the Discordancy measure test¹³ and Heterogeneity measure test¹⁴ have been selected.

Discordancy Measure

In this test, L -moment ratio (L -coefficient of variation ($L-C_v$), L -skewness, and L -kurtosis) of a site is used to describe that site in three-dimensional space. A group of homogeneous sites may form a cluster of such points. A point, which is far from the centre of the cluster, can be discarded. Discordancy measure of a site can be calculated by D_i , which is:

$$D_i = \frac{N}{3(N-1)} (u_i - \bar{u})^T S^{-1} (u_i - \bar{u}) \quad (5)$$

Generally any site with $D_i > 3$ can be regarded as discordant.

Heterogeneity Measure (H)

To compare the inter-site variation in sample L -moments for the group of sites with what would be expected of a homogeneous region, a test statistic H termed as heterogeneity measure is used. The inter-site variation of L -moment ratio is measured as the standard deviation (V) of at-site $L-C_v$'s weighted proportionally to the record length at each site. A number of, say 1000, data regions are generated based on the regional weighted statistics using a four parameter distribution, eg, Wakeby or kappa distribution.

The inter-site variation of each generated region is obtained and mean (μ_v) and standard deviation (σ_v) of the computed inter-site variation is determined. H can be estimated as:

$$H = \frac{V - \mu_v}{\sigma_v} \quad (6)$$

The value of H less than 1.0 indicates that the region is acceptably homogeneous. Similarly, the value of H between 1.0 to 2.0 indicates possibly homogeneous region. If, H is greater than 2.0, the region may be considered as heterogeneous.

Estimation of Mean Annual Peak Flood

For the ungauged catchment, the mean annual peak flood cannot be computed in the absence of flow data. In such a situation a relationship developed between mean annual peak flood and catchment characteristics such as catchment area, density and distribution of streams, overland and channel slopes and catchment storage from the gauged basins is used. In the present study, only catchment area has been considered because of lack of data and for sake of simplicity.

Flood Frequency Modelling

In the present study, approach based on at-site regional and regional only flood frequency modelling using standardized L -moments have been applied. The EV-I, GEV, logistic, generalized logistic, generalized pareto, normal and lognormal distributions have been fitted.

Selection of the Best-fit Distribution

In the flood frequency modelling, the goodness of fit tests measure whether the data of a particular site are consistent with the fitted probability distribution function to that site or not. The L -moment ratio diagram, measure based on L -kurtosis and Z -statistics have been used for selecting the best-fit-distribution.

L -Moment Ratio Diagram

The L -moment ratio diagram is a graph between L -kurtosis and L -skewness and can be used to select a suitable probability distribution for a region. Theoretical curves of various distributions as well as the regional L -skewness and L -kurtosis are plotted on same graph to select the best-fit-distribution.

Measure based on L -Kurtosis

Hosking and Wallis¹³ proposed a convenient method for goodness of fit based on L -kurtosis. In this test, the quality of fit is judged by the difference between the L -kurtosis of fitted distribution and the average L -kurtosis of the region. If σ_4 denotes the standard deviation of τ_4 , the good of fit measure, (say, for GEV distribution) is

$$Z^{GEV} = \frac{\tau_4 - \tau_4^{GEV}}{\sigma_4} \quad (7)$$

σ_4 can be obtained by repeated simulation of a homogeneous region with GEV distribution. The Z value of other distributions can be found by using parameters of these distributions in simulation.

Z -statistics

The Z -statistics (Z_i^{dist}) judged how well the L -moment ratios of the fitted distribution match with the regionally averaged

L-moment ratios. The Z_i^{dist} (Z^{LCV} , Z^{LCK} and Z^{LCS}) can be written as:

$$Z_i^{dist} = \frac{\bar{\tau}_i^R - \tau_i^{dist}}{\sigma_i^{dist}} \quad (8)$$

where $\bar{\tau}_i^R$ is the weighted regional average of L-moment statistic i ; τ_i^{dist} , simulated regional average of L-moment statistic i ; and σ_i^{dist} , the simulated regional standard deviation of L-moment statistic i .

The distribution giving the minimum $|Z^{dist}|$ is considered the best-fit distribution.

ANALYSIS AND RESULTS

Homogeneity Test

Discordancy measure for each site of the basin have been calculated and found between 0.114 to 0.521. The values of different heterogeneity measures H_{LCV} , H_{LCK} , H_{LCS} are found 0.364, 0.524 and 0.942, respectively. The results of both the tests confirm that the Beas basin may be considered as homogeneous region.

Estimation of Mean Annual Peak Flood

A relationship between mean annual peak flood (\bar{Q} , cumecs) and catchment area (A , km²) for the study area has been found as:

$$\bar{Q} = 0.659 \times A^{0.895} \quad (9)$$

A graphical representation of relationship between catchment area and mean annual flood has been presented in Figure 2.

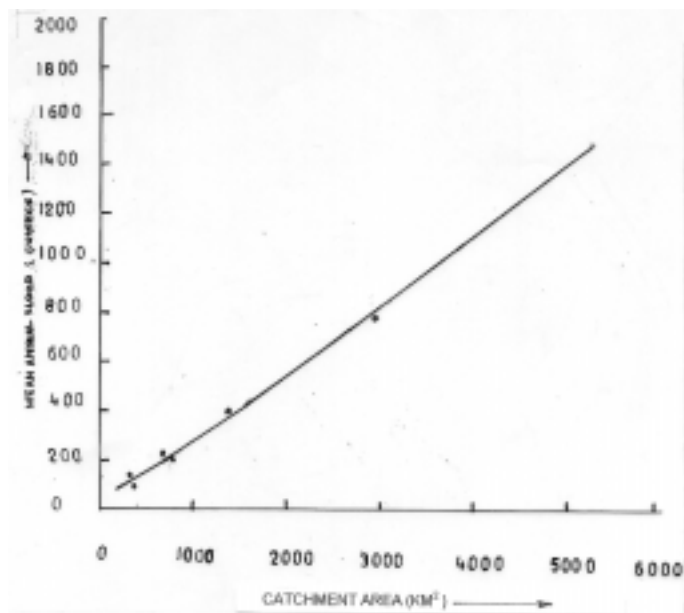


Figure 2 Graph showing relationship between mean annual flood and catchment area

Selection of the Best-fit Distribution

Parameters of different distributions based on L-moments have been computed and given in Table 2. The L-moment ratio diagram for the Beas basin has been presented in Figure 3. The results obtained from goodness of fit test based on L-kurtosis and Z-statistics are given in Tables 3 and 4, respectively.

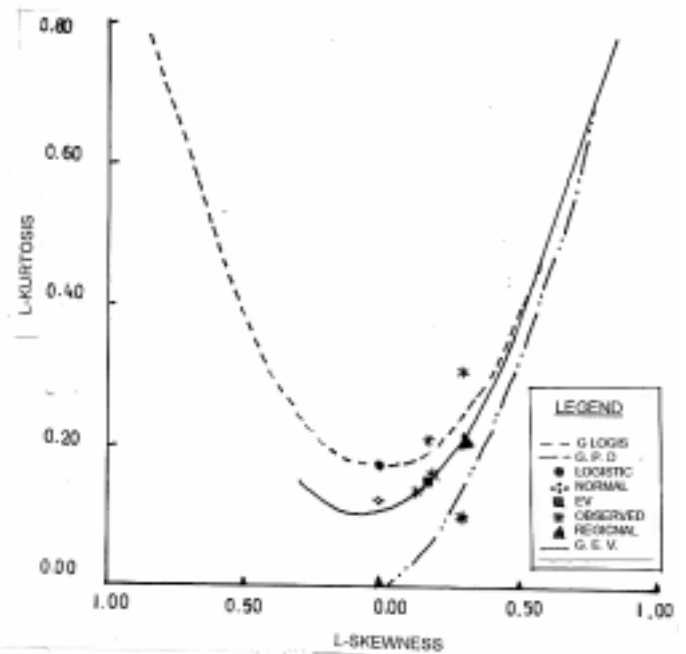


Figure 3 L-moment ratio diagram for some distributions and sites in the basin

Table 2 Parameters of various distributions used for analysis

L-moments			
λ_1	λ_2	λ_3	λ_4
1.00	0.254	0.07	0.044
L-moments Ratios			
L-CV		L-skewness	L-kurtosis
0.254		0.267	0.188
EV-I Distribution			
$\mu = 0.788$		$\alpha = 0.367$	
Generalised Extreme Distribution			
$\mu = 0.766$		$\alpha = 0.314$	$k = -0.147$
Logistic Distribution			
$\mu = 1.00$		$\alpha = 0.254$	
Generalised Logistic Distribution			
$\mu = 0.892$		$\alpha = 0.255$	$k = -0.267$
Normal Distribution			
$\mu = 1.000$		$s = 0.450$	
Lognormal Distribution			
$\mu = -0.338$		$s = 0.550$	$u = 0.618$

Table 3 Results of goodness of fit test based on *L*-kurtosis

Distribution	Z
EV-I	1.250
GEV	0.260
Logistic	0.715
Generalised Logistic	1.240
Generalised Pareto	2.299
Normal	2.163
Lognormal	2.256

Table 4 Z-statistics of different distributions used for flood frequency modelling

Distributions	Z _{LCV}	Z _{LCK}	Z _{LCS}
EV-I	1.025	1.011	1.010
GEV	0.410	0.131	0.160
Wakeby-4	0.311	0.635	0.537
Logistic	3.083	0.517	1.830
Generalised Logistic	2.648	0.312	1.228
Generalised Pareto	1.822	1.160	1.245
Normal	3.018	0.882	2.225
Lognormal	4.787	0.227	1.238

From the analysis of results of goodness of fit tests, it has been found that the GEV distribution can be considered as the best-fit distribution for the Beas basin up to Pandoh. The regional parameters of GEV distribution are: $\mu = 0.766$, $\alpha = 0.314$ and $k = -0.147$.

Flood Quantiles Estimation

Flood quantile (Q_T , cumecs) for a return period (T , years) can be computed using the relationship:

$$Q_T = \left[-1.37 + 2.136 \left\{ -\ln \left(1 - \frac{1}{T} \right) \right\}^{-0.147} \right] \times \bar{Q} \quad (10)$$

Mean annual flood (\bar{Q}) may be computed using equation (9) for a ungauged site. In case of gauged site, it should be computed from the observed data.

CONCLUSIONS

Flood frequency modelling is one of the simplest and widely used applications of statistics in the field of hydrology.

The *L*-moments have been used for parameter estimation, homogeneity testing and selection of the regional distribution in the study.

The GEV distribution can be considered as the most suitable distribution for the Beas basin. The flood quantiles for different return periods at any ungauged site in the Beas basin can be estimated with the help of equations (9) and (10).

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