

# Reliability Indices for 2-Component Non-identical System

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*This paper deals with the determination of reliability characteristics, eg, reliability function and mean time between failures (MTBF) which reveals the important measures of the effectiveness of the system in the reliability analysis. The model considers the 2-component non-identical system, which is under the influence of lethal shocks in addition to individual failures as well as non-lethal common cause shock failures. The emphasis is on the Markovian approach in order to derive the reliability characteristics which are mentioned above. Also, these measures are compared in the presence of lethal common cause shock failures with that of the situation when only individual failures are affecting the system. The numerical illustration is also discussed.*

**Keywords :** Mean time between failures (MTBF); Reliability; Markov approach

## NOTATION

$\lambda_0, \lambda_1$	: rates of failure of the first and second components respectively.
$\lambda_{12}$	: rate of common mode failure
$\mu_0, \mu_1$	: repair rates of first and second components, respectively
$\beta$	: rate of NCCS failure
$\omega$	: rate of LCCS failure
$R_{LS}^*(t)$	: reliability of the system in the case of LCCS model
$E_{LS}^*(T)$	: MTBF of the system in the case of LCCS model

## INTRODUCTION

The common cause shock failures are identified to be one of the most dominant causes of failures, which can severely degrade the actual operating system reliability. These are purely external causes, which effect multiple failures. For instance, two computers in one of the space — shuttle flights were repeatedly and simultaneously inoperable, because of jarring during operation of the space — shuttle. Many of such examples were seen in nuclear power plants, communication network and other such real applications.

Billinton and Allan<sup>1</sup> considered the Markovian approach for common cause failures in order to derive system reliability indices. Chari<sup>2,3</sup> derived the formulae for reliability indices under the influence of chance, lethal and non-lethal common cause shock failures. Verma<sup>4</sup> developed a model for estimating the reliability indices in the presence of common cause shock failures in addition to individual s - independent failures and the model assumes both identical and non-identical components.

The purpose of this paper is to derive the reliability of the system and mean time between failures with non-identical two components in the presence of lethal common cause shock (LCCS)

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failures. Therefore with certain probabilities, and hence is named the lethal common cause shock failures model (LCCS model). The impact of CCS failures on non-identical two component system is also established with numerical illustrations.

## LETHAL COMMON CAUSE SHOCK FAILURE MODEL

### Assumptions

- (i) The system has two S-independent and non-identical components.
- (ii) The components fail individually (or) simultaneously (LCCS failures).
- (iii) The random number of components fails due to non-lethal common cause shock, which is occurring at the rate  $\beta$ . Time to occurrence of NCCS failures also follow exponential distribution having p d f
 
$$f(x, \beta) = \beta \exp(-\beta x) \quad X \geq 0, \beta > 0$$
- (iv) The individual failures, non-lethal and lethal common cause shock failures occur independently with each other.
- (v) The rate of lethal shock occurring is ' $\omega$ '.
- (vi) The components are repaired singly.

Under the assumptions stated the markov graph for reliability analysis is seen in the Figure (1).

State spaces are defined as

- |           |  |
|-----------|--|
| State '0' | — Both components are working                                      |
| State '1' | — First component is not working while second component is working |
| State '2' | — First component is working while second component is not working |
| State '3' | — Both components are now working                                  |

The quantities that appear in the Figure (1) are to be read

$$\lambda_0 \text{ as } \lambda_1 + \beta p_1 (1 - p_2)$$

$$\lambda_1 \text{ as } \lambda_2 + \beta p_2 (1 - p_1)$$

$$\lambda_{12} \text{ as } \beta p_1 p_2 + \omega$$

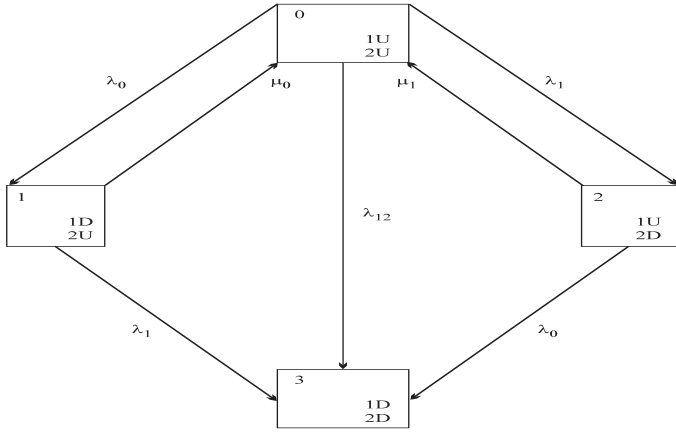


Figure 1 Markov graph of two component-non-identical system - LCCS model

The following markovian equations that result in the present model is as

$$\begin{aligned}
 \dot{P}_0(t+dt) &= P_0(t) \left[ 1 - \left\{ (\lambda_1 + \beta p_1 (1-p_2)) + (\lambda_2 + \beta p_2 (1-p_1)) + \beta p_1 p_2 + \omega \right\} dt + p_1(t) \mu_0 dt + p_2(t) \mu_1 dt \right] \\
 \dot{P}_1(t+dt) &= P_0(t) \left[ \lambda_1 + \beta p_1 (1-p_2) \right] dt + \left[ 1 - (\mu_0 + \{\lambda_2 + \beta p_2 (1-p_1)\}) dt \right] p_1(t) \\
 \dot{P}_2(t+dt) &= P_0(t) \left[ \lambda_2 + \beta p_2 (1-p_1) \right] dt + \left[ 1 - \{(\lambda_1 + \beta p_1 (1-p_2)) + \mu_1\} dt \right] p_2(t) \\
 \dot{P}_3(t+dt) &= P_0(t) (\beta p_1 p_2 + \omega) dt + \left[ (\lambda_2 + \beta p_2 (1-p_1)) dt p_1(t) + p_2(t) \right] \\
 &\quad \left[ \lambda_1 + \beta p_1 (1-p_2) \right] dt + p_3(t)
 \end{aligned} \tag{1}$$

On simplification the set of equations in (1) reduces to the following differential equations

$$\begin{aligned}
 \dot{P}_0^1(t) &= [ - (\lambda_1 + \beta p_1 (1-p_2)) + \lambda_2 + \beta p_2 (1-p_1) + (\beta p_1 p_2 + \omega) ] p_0(t) + \mu_0 p_1(t) + \mu_1 p_2(t) \\
 \dot{P}_1^1(t) &= [ \lambda_1 + \beta p_1 (1-p_2) ] p_0(t) - [ \lambda_2 + \beta p_2 (1-p_1) + \mu_0 ] p_1(t) \\
 \dot{P}_2^1(t) &= [ \lambda_2 + \beta p_2 (1-p_1) ] p_0(t) - [ \lambda_1 + \beta p_1 (1-p_2) + \mu_1 ] p_2(t) \\
 \dot{P}_3^1(t) &= (\beta p_1 p_2 + \omega) p_0(t) + [ \lambda_2 + \beta p_2 (1-p_1) ] p_1(t) + [ \lambda_1 + \beta p_1 (1-p_2) ] p_2(t)
 \end{aligned} \tag{2}$$

Using the method of Laplace transforms, the set of equations in (2) can be solved with the help of the initial conditions, given at  $t = 0$ ,  $p_0(t) = 1$  and  $p_0(t) = p_1(t) = p_1(t) = p_2(t) = p_3(t) = 0$  and the solution is

$$\begin{aligned}
 P_0(t) &= ((\gamma_1^2 + \gamma_1 k_1 + k_2)/(\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)) \exp(\gamma_1 t) \\
 &\quad - ((\gamma_2^2 + \gamma_2 k_1 + k_2)/(\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)) \exp(\gamma_2 t) \\
 &\quad + ((\gamma_3^2 + \gamma_3 k_1 + k_2)/(\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)) \exp(\gamma_3 t)
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 P_1(t) &= (\lambda_0 (\gamma_1 + \lambda_0 + \mu_1)/(\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)) \exp(\gamma_1 t) \\
 &\quad - (\lambda_0 (\gamma_2 + \lambda_0 + \mu_1)/(\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)) \exp(\gamma_2 t) \\
 &\quad + (\lambda_0 (\gamma_3 + \lambda_0 + \mu_1)/(\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)) \exp(\gamma_3 t)
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 P_2(t) &= (\lambda_1 (\gamma_1 + \lambda_1 + \mu_0)/(\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)) \exp(\gamma_1 t) \\
 &\quad - (\lambda_1 (\gamma_2 + \lambda_1 + \mu_0)/(\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)) \exp(\gamma_2 t) \\
 &\quad + (\lambda_1 (\gamma_3 + \lambda_1 + \mu_0)/(\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)) \exp(\gamma_3 t)
 \end{aligned} \tag{5}$$

$$P_3(t) = 1 - [P_0(t) + P_1(t) + P_2(t)] \tag{6}$$

where

$$\begin{aligned}
 \gamma_1 &= -r \sin \alpha - A/3 \\
 \gamma_2 &= r \sin(\pi/3 + \alpha) - A/3 \\
 \gamma_3 &= r \sin(-\pi/3 + \alpha) - A/3
 \end{aligned} \tag{7}$$

and  $\alpha = \theta$  is the solution of the equation,

$$\sin 3\theta = -4q/r^3$$

where  $q$  and  $r$  are given by

$$q = c - (AB/3) + (2A^3/27)$$

$$r = (2/3) \sqrt{A^2 - 3B}$$

and  $A, B, C, K_1$  and  $K_2$  are defined as,

$$\begin{aligned}
 A &= 2 \left\{ \lambda_1 + \beta p_1 (1-p_2) + \lambda_2 + \beta p_2 (1-p_1) \right\} + \beta p_1 p_2 + \omega + \mu_0 + \mu_1 \\
 B &= (\lambda_1 + \beta p_1 (1-p_2)) [\lambda_1 + \beta p_1 (1-p_2) + 3(\lambda_2 + \beta p_2 (1-p_1)) + \beta p_1 p_2 + \omega + \mu_0 + \mu_1] \\
 &\quad + (\lambda_2 + \beta p_2 (1-p_1)) [\lambda_2 + \beta p_2 (1-p_1) + \beta p_1 p_2 + \omega + \mu_0 + \mu_1] + (\beta p_1 p_2 + \omega) (\mu_0 + \mu_1) + \mu_0 \mu_1 \\
 C &= (\lambda_1 + \beta p_1 (1-p_2)) (\lambda_2 + \beta p_2 (1-p_1)) [\lambda_1 + \beta p_1 (1-p_2) + \lambda_2 + \beta p_2 (1-p_1) + \beta p_1 p_2 + \omega + \mu_0 + \mu_1] \\
 &\quad + (\beta p_1 p_2 + \omega) \mu_0 \left\{ \lambda_1 + \beta p_1 (1-p_2) + \mu_1 \right\} + (\lambda_2 + \beta p_2 (1-p_1)) (\beta p_1 p_2 + \omega) \mu_1
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 K_1 &= \lambda_1 + \beta p_1 (1-p_2) + \lambda_2 + \beta p_2 (1-p_1) + \mu_0 + \mu_1 \\
 K_2 &= \left\{ \lambda_2 + \beta p_2 (1-p_1) + \mu_0 \right\} \left\{ \lambda_1 + \beta p_1 (1-p_2) + \mu_1 \right\}
 \end{aligned} \tag{9}$$

$\gamma_1, \gamma_2$  and  $\gamma_3$  are always negative  $\forall \lambda_1, \lambda_2 \geq 0, p_1, p_2 \in [0, 1]$

## RELIABILITY OF THE SERIES SYSTEM - LCCS FAILURE MODEL

If the two component non-identical system under consideration is a series one, then the states one, two and three are considered to be dead states and hence transition from states one, two and three to zero state do not exist

$$ie (\mu_0 = \mu_1 = 0)$$

Therefore, the reliability of the series system for the LCCS model is

$$R_{LS}^*(t) = l_1 \exp(\gamma_1 t) - l_2 \exp(\gamma_2 t) + l_3 \exp(\gamma_3 t) \quad (10)$$

where,

$$\left. \begin{aligned} l_1 &= (\gamma_1^2 + \gamma_1 k_1 + k_2)/(\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3) \\ l_2 &= (\gamma_2^2 + \gamma_2 k_1 + k_2)/(\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3) \\ l_3 &= (\gamma_3^2 + \gamma_3 k_1 + k_2)/(\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3) \end{aligned} \right\} \quad (11)$$

$\gamma_1, \gamma_2$  and  $\gamma_3$  in the above equation are seen in (7). The quantities that appear in the expressions (10) and (11) are to be seen as,

$$A = 2 \{ \lambda_1 + \beta p_1 (1 - p_2) + \lambda_2 + \beta p_2 (1 - p_1) \} + \beta p_1 p_2 + \omega$$

$$B = (\lambda_1 + \beta p_1 (1 - p_2)) [\lambda_1 + \beta p_1 (1 - p_2) + 3 (\lambda_2 + \beta p_2 (1 - p_1)) + \beta p_1 p_2 + \omega] + (\lambda_2 + \beta p_2 (1 - p_1)) [\lambda_2 + \beta p_2 (1 - p_1) + \beta p_1 p_2 + \omega]$$

$$C = (\lambda_1 + \beta p_1 (1 - p_2)) (\lambda_2 + \beta p_2 (1 - p_1)) [\lambda_1 + \beta p_1 (1 - p_2) + \lambda_2 + \beta p_2 (1 - p_1) + \beta p_1 p_2 + \omega]$$

$$K_1 = \lambda_1 + \beta p_1 (1 - p_2) + \lambda_2 + \beta p_2 (1 - p_1)$$

$$K_2 = \{ \lambda_1 + \beta p_1 (1 - p_1) \} \{ \lambda_2 + \beta p_2 (1 - p_1) \}$$

The formula for reliability of the series system in the case of LCCS failures agree with the result of CHARI<sup>2</sup> for identical two component system by letting  $\lambda_1 = \lambda_2 = \lambda$  and  $P_1 = P_2 = P$  in expression (10) (using L Hospitals Rule). Also the expression agrees with the individual failures<sup>5</sup> in the case of identical components, if it is assumed that the LCCS as well as NCCS failures do not occur (ie,  $\beta = \omega = 0$ ).

## MTBF OF THE SERIES SYSTEM - LCCS FAILURE MODEL

For two components non-identical series system, the expected life (or) the average time during which an item performs its function successfully in the case of LCCS failures model is derived as

$$\begin{aligned} E_{LS}^*(T) &= \int_0^{\infty} R_{LS}^*(t) dt \\ &= \int_0^{\infty} [l_1 \exp(\gamma_1 t) - l_2 \exp(\gamma_2 t) + l_3 \exp(\gamma_3 t)] dt \\ &= (l_2/\gamma_2) - (l_1/\gamma_1) + (l_3/\gamma_3) \end{aligned} \quad (12)$$

Table 1 Reliability values for different sets of  $\beta$  and  $\omega$  as a function of time 't' - CCS model - series system

Time, hrs	$\beta = 0.0$ $\omega = 0.0$	$\beta = 0.1$ $\omega = 0.1$	$\beta = 0.2$ $\omega = 0.2$	$\beta = 0.3$ $\omega = 0.3$	$\beta = 0.4$ $\omega = 0.4$
0	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9704	0.8721	0.7837	0.7043	0.6329
2	0.9418	0.7606	0.6142	0.4961	0.4006
3	0.9139	0.6633	0.4814	0.3494	0.2536
4	0.8869	0.5785	0.3773	0.2461	0.1606
5	0.8607	0.5045	0.2957	0.1734	0.1017
6	0.8353	0.4400	0.2318	0.1222	0.0644
7	0.8106	0.3837	0.1817	0.0861	0.0408
8	0.7866	0.3346	0.1424	0.0607	0.0259
9	0.7634	0.2919	0.1117	0.0428	0.0165
10	0.7408	0.2545	0.0875	0.0302	0.0105

where  $l_1, l_2$  and  $l_3$  are seen to be in the expression (11) and  $\gamma_1, \gamma_2$  and  $\gamma_3$  are defined as seen in equation (7).

The result obtained in expression (12) will agree with the one already developed<sup>1</sup> for the individual failure cases when it is assumed that no LCCS as well as NCCS failures are acting on the system (ie,  $\beta = \omega = 0$ ).

## NUMERICAL RESULTS

The values of reliability in the series configuration is obtained as a function of time 't' taking the failure rates as  $\lambda_1 = 0.01$  failures/h,  $\lambda_2 = 0.02$  failures/h, with  $p_1 = 0.03$  and  $p_2 = 0.04$  for various values of lethal and non-lethal common cause shock failures, and are shown in Table 1.

## CONCLUSION

From the numerical work it was observed that the reliability of the two - component non-identical system in the case of series configuration is less for the LCCS failures model than for the case with individual failures (ie,  $\beta = \omega = 0$ ). Hence the present expression may be used to assess the reliability when the system is affected by CCS failures in addition to individual failures. For the LCCS failures model, when the rate of CCS failures (lethal and non-lethal common - cause shock failures) increases, the system reliability decreases rapidly. This necessitates the inclusion of CCS failures when the system is under the influence of CCS failures, to assess correctly the reliability indices.

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