

Finite Element Analysis and Reliability Assessment of Spherical LPG Storage Tank

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The present study deals with finite element analysis (FEA) and reliability assessment of spherical LPG storage tank. Finite element analysis is carried out to determine the maximum deflection and stress distribution in the tank. The shell and beam materials are assumed to follow the Von-Mises yield criterion. The elastic buckling of spherical storage tank is considered in this study. Reliability assessment against internal pressure, wind and earthquake loading has been carried out using first order reliability method. Sensitivity analysis has also been performed by perturbing different parameters and by noting, the corresponding variation in the stress developed.

Keywords: FEA; Reliability assessment; Spherical storage tank; Probability of failure; Buckling

NOTATION

A	: projected area	$g(z)$: performance function
$\{B\}$: strain matrix	h_c	: height of the column
$[B]$: gradient matrix	I	: importance factor
C	: base factor	K	: arrangement factor
cov	: coefficient of variance	$\{K_e\}$: stiffness matrix an element
C_f	: pressure coefficient	$\{K\}_g$: global stiffness matrix
d_i	: inner diameter of the column	K_z	: exposure coefficient
d_o	: outer diameter of the column	$[N_u]_e$: shape function matrix
$\{D\}$: material property matrix	p	: internal pressure
D_i	: inner diameter of spherical shell	P_f	: probability of failure
D_o	: outer diameter of spherical shell	S	: site structure resonance
$\{D_T\}$: tangential elasticity matrix	t	: thickness of the shell
E	: Young's modulus	$\{U\}$: displacement vector
E_e	: gust factor	$\{U_N\}_e$: nodal displacement matrix
$\{f\}_e$: force matrix for an element	V	: volume of shell
$[f]_g$: global force matrix	V_w	: velocity of wind
f_{ys}	: yield stress of the material	W_a	: wind load
$\{F\}$: nodal/elemental force	W_e	: earthquake load
F_{ys}	: Poisson's ratio	Z	: earthquake zone coefficient
		β	: reliability index
		μ_L	: mean value of load
		μ_R	: mean value of resistance
		ρ	: density of the shell material
		ρ_a	: density of air

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This paper (modified) was received on December 12, 2002. Written discussion on this paper will be entertained till December 31, 2003.

- $\{\sigma\}_e$: matrix for stress tensor
- σ_L : standard deviation of load
- σ_R : standard deviation of resistance
- $\{\sigma_\phi\}$: meridonal direct stress
- $\Phi(\beta)$: cumulative probability distribution function

INTRODUCTION

The spherical pressure vessels are used extensively to store the liquid chlorine, anhydrous ammonia, sulphur dioxide, ethylene oxide, carbon dioxide, etc. These vessels are used in paper and pulp mills, caustic soda plants, bleaching, plants for water purification, sewage treatment plants, refineries, fertilizer industry, and in PVC plants.

Stress analysis problems of pressure vessels entail two types of considerations, *ie*, applied loads and the corresponding response. A realistic way to tackle the first consideration is to design the loads, which have a high probability of occurring during a rigorously specified usage of the vessel. There are two well-tried types of techniques for handling the second consideration, predicting the response of the structure to specified loads. The first type of technique is model analysis. The response of the model is measured and used to interpret the behaviour of the prototype structure. The second technique is via the solution of the vessels governing equations. Elastic problems are governed by linear differential equations. More complex material behaviour can often be approximated by a sequence of simpler behaviours, *eg*, by subdividing its time regime into a sequence of time increments.

The stress analysis of pressure vessel is carried out by using finite element software. In the present paper, linear static and buckling analysis have been conducted. In static analysis Von-Misses stress, resultant displacements and resultant deformation have been observed. In buckling analysis, the buckling load factors and maximum deflection for various mode shape have been observed.

FINITE ELEMENT FORMULATION

In finite element method, displacement approach is normally adopted, in which one has to proceed as follows¹.

Assume the variation of displacements $\{u\}_e$ within an element as

$$\{u\}_e = [N_u]_e \{U_N\}_e \quad (1)$$

Now using some variational principal or by applying weighted residual technique to equilibrium equations of elasticity, following equation is obtained.

$$\int [B]^T \{\sigma\}_e dv = \{f\}_e \quad (2)$$

Since, stresses are related to displacements

$$\{\sigma\}_e = [D][B]\{U_N\}_e \quad (3)$$

where $[D]$ relates stresses to strains. Substituting equation (3) into equation (2).

$$\int [B]^T [D][B] dv \{U_N\}_e = \{f\}_e \quad (4)$$

or

$$[K]_e \{U_N\}_e = \{f\}_e \quad (5)$$

These element level equations can be assembled and the global stiffness matrix $[K]_g$ and global force matrix $[f]_g$ can be framed.

$$[K]_g \{U_N\}_g = \{f\}_g \quad (6)$$

After putting known displacement boundary condition, the unknown displacements can be solved. After finding displacements, stresses are determined for an element using equation (3).

Type of Elements Used

While using EMRC-NISA FEM software², 3-D general shell elements are used for spherical shell and 3-D beam elements are used for the columns. Shell element includes membrane; bending and transverse shear deformation effects and is suited for modelling moderately thick to thin shell structures. The element has six degree-of-freedom per node ($U_x, U_y, U_z, R_x, R_y, R_z$). The transverse shear stress is zero on the top and bottom surface and maximum at the middle surface. The element has eight nodes defined on its mid-surface, four on its corners and four on mid-side of each edge. A natural co-ordinate system is also defined on mid-surface. Both ξ and η vary between 1 and -1 and have zero values at mid-surface centre.

The final deformation in the element is described in terms of the corner nodes only. The middle surface mid-side nodes are used only to establish the appropriate geometrical quantities needed to define the shape of the element. It maintains the usual shell geometric properties and provides membrane and bending stresses separately. In this element, a continuous function representing the through thickness variation of stresses and displacements is used¹.

The 3D, 2-noded beam elements also have six degree-of-freedom ($U_x, U_y, U_z, R_x, R_y, R_z$) at each node and capable of taking shear effects into picture. The degree-of-freedom of shell and beam elements are same, hence they are well suited for such type of work, where beams are connected to shell.

Analysis Procedure

A detailed study of stress analysis of spherical pressure vessel with internal pressure, self-weight, wind load and earthquake load has been carried out with the help of FEM, which is perhaps the best currently known method available for the stress analysis of pressure vessel problems. The objective of this analysis is to find out the locations where stresses and displacements have maximum values, when subjected to the above mentioned load conditions.

Geometric model of the pressure vessel is constructed by rotation of meridian about an axis of rotation (in the present paper it is y -axis) in the plane of meridian (xy). Shells used in the vessel construction are considered as thin shells, since wall thickness is quite small in comparison with other dimensions. Stresses and displacements are calculated for working pressure and as well as for maximum designed pressures by means of NISA FEM software. For finite element modelling, total number of elements used are 1090 and total number of nodes used are 1864.

Element and Material Properties

3-D shell element:

Thickness of the shell, m : 0.03

3-D beam element:

Outer diameter, m : 0.62

Inner diameter, m : 0.58

Cross-sectional area, m^2 : 0.037699

Moment of inertia, m^4 ($I_{xx} = I_{yy}$) : 1.6983

Density, kg/m^3 : 7800

Young's modulus, N/m^2 : 206.01×10^9

Poisson's ratio : 0.28

Results

The maximum Von-Misses stresses against working and as well as for maximum pressure are given in Table 1.

Figure 1 shows the Von-Misses stresses in the storage tank. The maximum stress is well within the limits. Figure 2 shows the resultant deformation. Finally, the Figure 3 shows the displacements in Y direction. The maximum values of displacements in Y and X direction are 0.0056 m and 0.1167 m, respectively.

Table 1 The internal pressure and corresponding maximum Von-Misses stress

Internal Pressure, $N/m^2 \times 10^5$	Maximum Von-Misses Stress, $N/m^2 \times 10^5$
14.0	1751.172
6.5	867.200

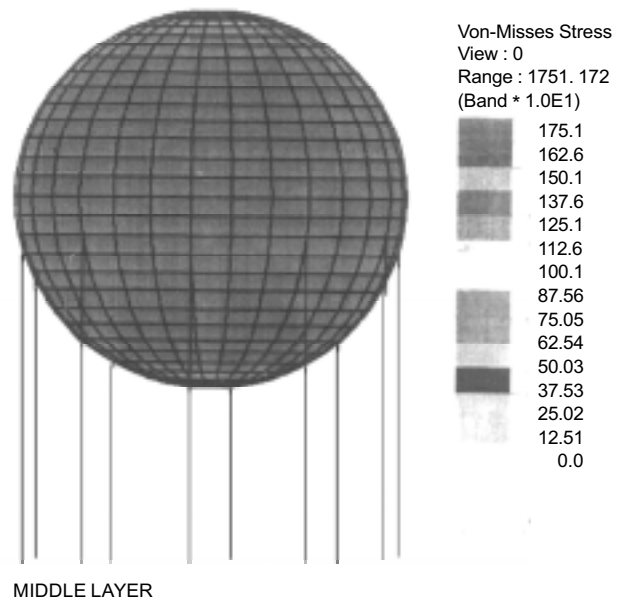


Figure 1 Von-Misses stress (front view)

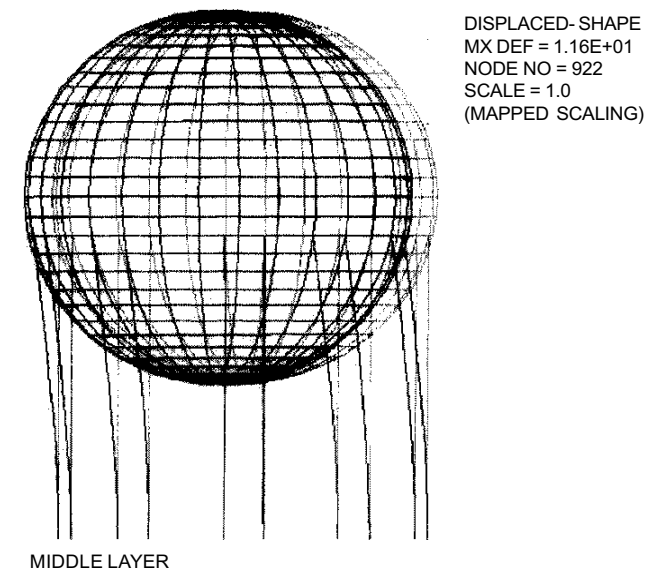


Figure 2 Resultant deformations

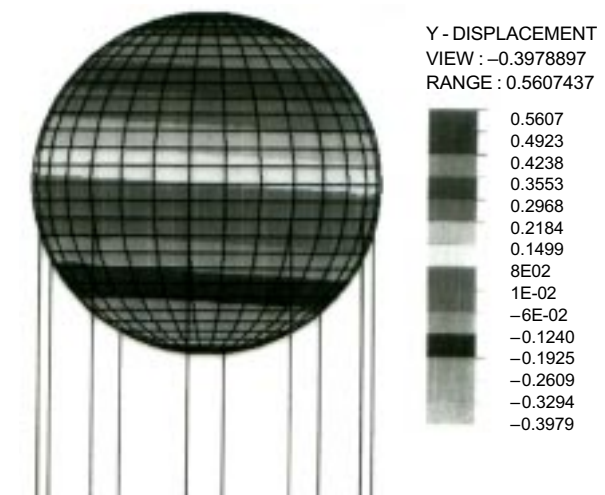


Figure 3 Displacements in Y-direction

BUCKLING ANALYSIS

The aim of the buckling analysis is to estimate the maximum load that a structure can support before it becomes elastically unstable or before it collapses. The observed buckling pressure depends on how buckling is defined. According to Galletly³ ‘the buckling pressure is defined as that pressure at which the first buckle becomes visible to the unaided eye’. In this paper, the analysis is carried out to examine the elastic buckling behaviour of spherical shells with respect to the geometric parameters. This will enable a designer to predict the elastic internal buckling pressures of sphere quickly, reasonably and accurately. It is assumed that the shells are of constant thickness and are free from imperfections.

Analysis Procedure

In NISA, buckling analysis is a two-pass analysis, static analysis and eigen values analysis. The analysis is actually a bifurcation buckling. Since, it involves calculating the points at which the structure’s primary load deflection path, a straight line for a linear static analysis, is bifurcated by a secondary load, deflection path. Governing matrix equation for buckling analysis is a general eigen value problem.

$$[K]\{b\} = -\lambda[K_\sigma]\{b\} \quad (7)$$

$[K_\sigma]$ = geometric stiffness

λ = eigen value (load factor)

The buckling load factors are multipliers for the reference loads in the static pass. Internal pressure 137.34 N/m² is applied as a reference load for doing buckling analysis.

Results for Buckling of Spherical Pressure Vessel

Ten different mode shapes and corresponding buckling load factors are obtained. The values of buckling load factor and maximum deflection for these mode shapes are given in Table 2.

Table 2 The mode shape and corresponding buckling factor

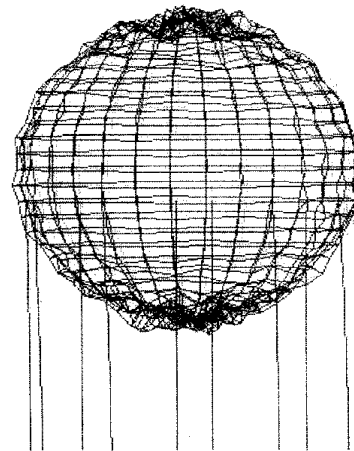
Mode Shape	Buckling Factor (- ve)	Maximum Deflection × 10 ⁻³
1	7.52397	4.092
2	6.18975	3.526
3	6.07652	1.832
4	5.87589	1.866
5	5.72907	1.421
6	5.06189	2.097
7	5.49206	2.004
8	5.44479	1.676
9	5.27776	1.730
10	5.32453	1.873



DISPLACED SHAPE
MX DEF = 4.09E-03
NODE NO = 774
SCALE = 1.0
(MAPPED SCALING)

MODE NO 1 BUCKLING LOAD FACTOR = -7.52397E + 00

Figure 4 First mode shape with buckling load factors



DISPLACED SHAPE
MX DEF = 1.42E-03
NODE NO = 929
SCALE = 1.0
(MAPPED SCALING)

MODE NO 6 BUCKLING LOAD FACTOR = -5.72907E + 00

Figure 5 Fifth mode shape with buckling load factors

The Figure 4 and Figure 5 show the first and fifth mode shapes, respectively and the corresponding buckling load factors. From Table 2, it is observed that the buckling load factor for the lower mode shapes is coming higher.

RELIABILITY ASSESSMENT

The reliability of an engineering system can be defined as its ability to fulfil its design purpose satisfactorily all through its service period. The theory of statistics and probability provides the mathematical basis to measure this ability. For the present problem, the Von-Misses criteria are considered as the failure criteria and first order second moment method FOSM or MVFOSM as the reliability method⁴. The FOSM is quite simple to apply and computationally less expensive than other methods and gives reasonably good results. That is why for the present problem this method is selected.

The FOSM or MVFOSM method derives its name from the fact that it is based on a first order Taylor series approximation of the limit state function at the mean values of the random

variables. Expansion of limit state function $g(\underline{z})$ in a Taylor series about the mean values of z_i

$$g(\underline{z}) = g(z_1, z_2, z_3, \dots, z_i) \quad (8)$$

= structural strength – load effect

$$g(\underline{z}) = g(\bar{z}) + \sum_{i=1}^n \frac{\partial g}{\partial z_i} (z_i - \bar{z}_i) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g}{\partial z_i \partial z_j} \times (z_i - \bar{z}_i)(z_j - \bar{z}_j) + \dots \quad (9)$$

Truncating the series and retaining only the linear terms, the first order approximate mean and variance of $g(\underline{z})$ are $\mu_g = g(\bar{z}_1, \bar{z}_2, \bar{z}_3, \dots, \bar{z}_n)$.

$$\sigma_g^2 = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial g}{\partial z_i} \frac{\partial g}{\partial z_j} \text{cov}(z_i - z_j) \quad (10)$$

where $\text{cov}(z_i - z_j)$ is the covariance of z_i and z_j ; μ_g , the mean value of g and is variance of $g(\underline{z})$. The partial derivative has been evaluated at the mean value of the basic random variables.

$$\sigma_g^2 = \sum_{i=1}^n \left(\frac{\partial g}{\partial z_i} \right)^2 \text{var}(z_i) \quad (11)$$

where $\text{var}(z_i)$ is the variance of z_i .

Reliability Index

Reliability index β is a measure of reliability (safety) of a structure. It is defined as the ratio of mean and standard deviation of limit state function, *ie*,

$$\beta = \frac{\mu_z}{\sigma_z} \quad (12)$$

where μ_z and σ_z are the mean value and standard deviation of limit state function $g(\underline{z})$. β is the reciprocal of the coefficient of variation of $g(\underline{z})$, *ie*, $\beta = 1/(\sigma_z / \mu_z)$. The safety is defined by the condition $g(\underline{z}) > 0$, therefore failure by $g(\underline{z}) < 0$.

Probability of Failure

Probability of failure is defined as the probability of limit state violation expressed through a limit state function, *ie*, probability of $g(\underline{z}) < 0$ is assumed to be normally distributed. It can be shown that the probability of failure $P_f = 1 - \Phi(\beta)$, where Φ is the cumulative probability distribution function of standard normal variate. The aforementioned MVFOSM produces accurate results when the random variables are normally distributed and the limit state function $g(\underline{z})$ is linear.

Methodology

The problem under study contains the internal pressure, wind load and earthquake load as loads. So Von-Mises stress is the function of internal pressure, wind load, earthquake load, diameter and thickness of the pressure vessel.

$$f_v = f(P, w_w, w_c, D, t) \quad (13)$$

Table 3 Variation of Von-Mises stress with pressure and thickness

Parameter : Pressure and thickness
 Mean value : 140×10^4 N/m² and 0.03 m, respectively
 Percentage change in variation : 10% and 20%, respectively

Pressure, N/m ² × 10 ⁵	Maximum Von-Mises Stress, N/m ² × 10 ⁵	Thickness, m × 10 ²	Maximum Von-Mises Stress, N/m ² × 10 ⁵
13.30	1663.636	2.850	1848.354
13.44	1676.140	2.900	1815.125
13.58	1701.152	2.925	1798.352
13.72	1713.657	2.950	1782.376
13.86	1739.259	2.975	1767.408
14.00	1751.172	3.000	1751.172
14.14	1763.678	3.025	1735.992
14.28	1788.688	3.050	1721.073
14.42	1801.193	3.075	1706.408
14.56	1826.264	3.100	1691.992

Table 4 Variation of Von-Mises stress with wind load and earthquake load

Parameter : Wind load and earthquake load
 Mean value : 830 N per node and 395 N per node, respectively
 Percentage change in variation : 20% for both

Wind Load, × 10 N	Maximum Von-Mises Stress, × 10 ⁵ N/m ²	Earthquake Load, × 10 N	Maximum Von-Mises Stress, × 10 ⁵ N/m ²
74.70	1765.666	35.55	1746.433
76.36	1767.147	36.34	1751.171
78.02	1769.113	37.13	1751.504
79.68	1772.238	37.92	1751.883
82.34	1773.539	38.71	1753.528
83.00	1774.772	39.50	1759.127
84.66	1776.741	40.29	1765.408
86.32	1777.296	41.08	1777.508
87.98	1778.397	41.87	1779.409
89.64	1781.456	42.66	1779.730
91.30	1782.743	43.45	1781.466

The present analysis has been done by varying the individual parameters for number of time and the maximum Von-Misses stress for each and every time is obtained for individual parameters. The variation of Von-Misses stress with respect to individual parameters like internal pressure wind load, earthquake load, diameter and thickness of pressure vessel are given in Table 3 and Table 4.

Results for Reliability

From the individual study of parametric variation, the stress is mostly effected by the variation of the internal pressure. A linear variation is observed against change of thickness of the shell. A small variation in the Von-Misses stress is observed in case of wind load and earthquake load. But in the earthquake load, drastic increase of stress is observed at a load of 410 N per node, which can be concluded as a critical load.

$$\text{Reliability index } (\beta) = 2.464$$

$$\text{Probability of failure } (P_f) = 0.70\%$$

CONCLUSIONS

The working pressure of the vessel is 63.765 N/m² to 78.48 N/m² and the maximum design pressure is 137.34 N/m². The other loads, which are acting on the vessel, are wind load and earthquake load along with the self-weight of the vessel. Failure of the vessel is a matter of great concerns, which at the upper extreme involves bursting of the vessel and at lower extreme, involves the attainment by the stress intensity of some critical value. The maximum values of effective stresses and deflections have been determined. The maximum

effective Von-Misses stresses are near the junctions at the upstream side of the wind. It has been investigated that the pressure vessel is safe from buckling failure because buckling load factor is -7.52397.

From the reliability analysis, it has been shown that the vessel is 99.3% reliable and the safety or reliability index is 2.464, which is within the required limit between 2 to 5. It has also been observed that, the effect of earthquake and wind loads is quite less due to the reason that a heavy supporting frame of columns has been provided to avoid any catastrophic calamity.

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